

**EXERCICES CORRIGES : LIMITES ET CONTINUITÉ**

objectifs

- Limites à l'infinie d'expression avec radical
- Limites trigonométriques
- Utilisation du nombre dérivé

**Calculer les limites suivantes**

1.  $\lim_{x \rightarrow +\infty} \sqrt{4x^2 + x + 1}$

2.  $\lim_{x \rightarrow -\infty} \sqrt{4x^2 + x + 1}$

3.  $\lim_{x \rightarrow +\infty} \sqrt{\frac{x^3 - 1}{x + 2}}$

4.  $\lim_{x \rightarrow -\infty} \sqrt{\frac{x^3 - 1}{x + 2}}$

5.  $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1}}{x}$

6.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x}$

7.  $\lim_{x \rightarrow +\infty} \frac{1 - \sqrt{x^2 + 1}}{1 + \sqrt{x^2 + 1}}$

8.  $\lim_{x \rightarrow -\infty} \frac{1 - \sqrt{x^2 + 1}}{1 + \sqrt{x^2 + 1}}$

9.  $\lim_{x \rightarrow +\infty} \frac{x - 2\sqrt{x}}{3x - 1}$

10.  $\lim_{x \rightarrow \frac{1}{3}} \frac{x - 2\sqrt{x}}{3x - 1}$

11.  $\lim_{x \rightarrow +\infty} \frac{1 - \sin x}{x - \frac{\pi}{2}}$

12.  $\lim_{x \rightarrow 0} \frac{\sin(\tan x)}{\tan x}$

13.  $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$

14.  $\lim_{x \rightarrow +\infty} \frac{\sin\left(x - \frac{\pi}{3}\right)}{3x - \pi}$

15.  $\lim_{x \rightarrow +\infty} \left(\frac{x}{2} - 3\sqrt{x}\right)$

16.  $\lim_{x \rightarrow +\infty} \sqrt{4x^2 - 2} - 2x$

17.  $\lim_{x \rightarrow -\infty} \sqrt{4x^2 - 2} - 2x$

18.  $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{x+1} - 1}$

19.  $\lim_{x \rightarrow +\infty} \sqrt{x^2 + 1} - \sqrt{x^2 + x}$

20.  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

21.  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1+x}}{x}$

22.  $\lim_{x \rightarrow 2} \frac{1 - \sqrt{3x-2}}{\sqrt{2x+5} - 3}$

23.  $\lim_{x \rightarrow +\infty} \cos\left(\frac{x+1}{x+2}\right)$

24.  $\lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x}}$

25.  $\lim_{x \rightarrow 1} \cos \pi x + \frac{1}{x-1}$

26.  $\lim_{x \rightarrow \frac{\pi}{2}} \tan x - \frac{2}{x}$

27.  $\lim_{x \rightarrow 0} \frac{\sin x}{3x}$

28.  $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x}}$

29.  $\lim_{x \rightarrow 0} \frac{x}{\tan 2x}$

30.  $\lim_{x \rightarrow 0} \frac{\sin x}{1 - \cos x}$

31.  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

32.  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sqrt{1 - \cos x}}$

**Correction**

$$\begin{aligned}
 1. \lim_{x \rightarrow +\infty} \sqrt{4x^2 + x + 1} &= \lim_{x \rightarrow +\infty} \sqrt{x^2 \left( 4 + \frac{1}{x} + \frac{1}{x^2} \right)} = \lim_{x \rightarrow +\infty} |x| \sqrt{\left( 4 + \frac{1}{x} + \frac{1}{x^2} \right)} \\
 &= \lim_{x \rightarrow +\infty} x \sqrt{\left( 4 + \frac{1}{x} + \frac{1}{x^2} \right)} = +\infty, \text{ car } \frac{1}{x} \rightarrow 0 \text{ et } \frac{1}{x^2} \rightarrow 0
 \end{aligned}$$

$$\begin{aligned}
 2. \lim_{x \rightarrow +\infty} \sqrt{4x^2 + x + 1} &= \lim_{x \rightarrow +\infty} \sqrt{x^2 \left( 4 + \frac{1}{x} + \frac{1}{x^2} \right)} = \lim_{x \rightarrow +\infty} |x| \sqrt{\left( 4 + \frac{1}{x} + \frac{1}{x^2} \right)} \\
 &= \lim_{x \rightarrow +\infty} -x \sqrt{\left( 4 + \frac{1}{x} + \frac{1}{x^2} \right)} = +\infty, \text{ car } \frac{1}{x} \rightarrow 0 \text{ et } \frac{1}{x^2} \rightarrow 0
 \end{aligned}$$

$$3. \lim_{x \rightarrow +\infty} \sqrt{\frac{x^3 - 1}{x + 2}} = \lim_{x \rightarrow +\infty} \sqrt{\frac{x^3}{x}} = \lim_{x \rightarrow +\infty} \sqrt{x^2} = \lim_{x \rightarrow +\infty} |x| = +\infty$$

$$4. \lim_{x \rightarrow -\infty} \sqrt{\frac{x^3 - 1}{x + 2}} = \lim_{x \rightarrow -\infty} \sqrt{\frac{x^3}{x}} = \lim_{x \rightarrow -\infty} \sqrt{x^2} = \lim_{x \rightarrow -\infty} |x| = +\infty$$

$$5. \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1}}{x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 \left( 1 + \frac{1}{x} \right)}}{x} = \lim_{x \rightarrow +\infty} \frac{|x| \sqrt{\left( 1 + \frac{1}{x} \right)}}{x} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{\left( 1 + \frac{1}{x} \right)}}{x} = 1$$

$$6. \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 \left( 1 + \frac{1}{x} \right)}}{x} = \lim_{x \rightarrow -\infty} \frac{|x| \sqrt{\left( 1 + \frac{1}{x} \right)}}{x} = \lim_{x \rightarrow -\infty} \frac{-x \sqrt{\left( 1 + \frac{1}{x} \right)}}{x} = -1$$

$$7. \lim_{x \rightarrow +\infty} \frac{1 - \sqrt{x^2 + 1}}{1 + \sqrt{x^2 + 1}} = \lim_{X \rightarrow +\infty} \frac{1 - X}{1 + X} = -1, \text{ avec } X = \sqrt{x^2 + 1} \text{ et } \lim_{x \rightarrow +\infty} \sqrt{x^2 + 1} = +\infty$$

$$8. \lim_{x \rightarrow -\infty} \frac{1 - \sqrt{x^2 + 1}}{1 + \sqrt{x^2 + 1}} = \lim_{X \rightarrow +\infty} \frac{1 - X}{1 + X} = -1 \text{ avec } X = \sqrt{x^2 + 1} \text{ et } \lim_{x \rightarrow -\infty} \sqrt{x^2 + 1} = +\infty$$

$$9. \lim_{x \rightarrow +\infty} \frac{x - 2\sqrt{x}}{3x - 1} = \lim_{X \rightarrow +\infty} \frac{X^2 - 2X}{3X^2 - 1} = \lim_{X \rightarrow +\infty} \frac{X^2}{3X^2} = \frac{1}{3}$$

$$10. \lim_{x \rightarrow \frac{1}{3}} \frac{x - 2\sqrt{x}}{3x - 1} = -\infty, \text{ car } x - 2\sqrt{x} \text{ est négatif et } 3x - 1 \text{ tend vers } 0$$

$$11. \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-(\sin x - 1)}{x - \frac{\pi}{2}} = -\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}} = -\lim_{x \rightarrow \frac{\pi}{2}} \frac{f(x) - f\left(\frac{\pi}{2}\right)}{x - \frac{\pi}{2}} = -f'\left(\frac{\pi}{2}\right) \\ = \cos\left(\frac{\pi}{2}\right) = 0$$

$$12. \lim_{x \rightarrow 0} \frac{\sin(\tan x)}{\tan x} = \lim_{X \rightarrow 0} \frac{\sin X}{X} = 1, \text{ avec } X = \tan x \text{ et } \lim_{x \rightarrow 0} \tan x = 0$$

$$13. \lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x} = -\lim_{x \rightarrow \pi} \frac{\sin x - \sin \pi}{x - \pi} = \cos \pi = -1$$

$$14. \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin\left(x - \frac{\pi}{3}\right)}{3x - \pi} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin\left(x - \frac{\pi}{3}\right)}{3\left(x - \frac{\pi}{3}\right)} = \lim_{X \rightarrow 0} \frac{\sin X}{3X} = \frac{1}{3} \lim_{X \rightarrow 0} \frac{\sin X}{X} = \frac{1}{3}$$

$$15. \lim_{x \rightarrow +\infty} \left(\frac{x}{2} - 3\sqrt{x}\right) = \lim_{X \rightarrow +\infty} \left(\frac{X^2}{2} - 3X\right) = +\infty, \text{ avec } X = \sqrt{x}$$

$$16. \lim_{x \rightarrow +\infty} \sqrt{4x^2 - 2} - 2x = \lim_{x \rightarrow +\infty} \frac{(\sqrt{4x^2 - 2} - 2x)(\sqrt{4x^2 - 2} + 2x)}{\sqrt{4x^2 - 2} + 2x} \\ = \lim_{x \rightarrow +\infty} \frac{4x^2 - 2 - 4x^2}{\sqrt{4x^2 - 2} + 2x} = \lim_{x \rightarrow +\infty} \frac{-2}{\sqrt{4x^2 - 2} + 2x} = \lim_{x \rightarrow +\infty} \frac{-2}{2x\sqrt{\left(1 - \frac{1}{2x^2}\right)} + 2x} \\ = \lim_{x \rightarrow +\infty} \frac{-2}{2x\left(\sqrt{\left(1 - \frac{1}{2x^2}\right)} + 1\right)} = 0$$

$$17. \lim_{x \rightarrow -\infty} \sqrt{4x^2 - 2} - 2x = \lim_{x \rightarrow -\infty} \sqrt{(2x)^2 - 2} - 2x = \lim_{x \rightarrow -\infty} \sqrt{(2x)^2 \left(1 - \frac{2}{4x^2}\right)} - 2x$$

$$= \lim_{x \rightarrow \infty} |2x| \sqrt{\left(1 - \frac{2}{4x^2}\right)} - 2x = \lim_{x \rightarrow \infty} \left(-2x \sqrt{\left(1 - \frac{2}{4x^2}\right)} - 2x\right)$$

$$= \lim_{x \rightarrow \infty} -2x \left(\sqrt{\left(1 - \frac{2}{4x^2}\right)} + 1\right) = +\infty$$

$$18. \lim_{x \rightarrow +\infty} \sqrt{x^2 + 1} - \sqrt{x^2 + x} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1} - \sqrt{x^2 + x})(\sqrt{x^2 + 1} + \sqrt{x^2 + x})}{\sqrt{x^2 + 1} + \sqrt{x^2 + x}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 + 1 - x^2 - x}{\sqrt{x^2 + 1} + \sqrt{x^2 + x}} = \lim_{x \rightarrow +\infty} \frac{1 - x}{\sqrt{x^2 + 1} + \sqrt{x^2 + x}} = \lim_{x \rightarrow +\infty} \frac{1 - x}{\sqrt{x^2 \left(1 + \frac{1}{x^2}\right)} + \sqrt{x^2 \left(1 + \frac{1}{x}\right)}}$$

$$= \lim_{x \rightarrow +\infty} \frac{1 - x}{x \left(\sqrt{\left(1 + \frac{1}{x^2}\right)} + \sqrt{\left(1 + \frac{1}{x}\right)}\right)} = \lim_{x \rightarrow +\infty} \frac{1 - x}{x} \times \frac{1}{\left(\sqrt{\left(1 + \frac{1}{x^2}\right)} + \sqrt{\left(1 + \frac{1}{x}\right)}\right)} = -\frac{1}{2}$$

$$19. \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3} = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = f'(3)$$

$$\text{Où } f(x) = \sqrt{x+1} \quad f'(x) = \frac{1}{2\sqrt{x+1}} \quad f'(3) = \frac{1}{2\sqrt{3+1}} = \frac{1}{4}$$

$$20. \lim_{x \rightarrow 0} \frac{1 - \sqrt{1+x}}{x} = -\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = -\lim_{x \rightarrow 0} \frac{u(x) - u(0)}{x - 0} = u'(0)$$

$$\text{Où } u(x) = \sqrt{x+1} \quad u'(x) = \frac{1}{2\sqrt{x+1}} \quad u'(0) = \frac{1}{2\sqrt{0+1}} = \frac{1}{2}$$

$$21. \lim_{x \rightarrow 2} \frac{1 - \sqrt{3x-2}}{\sqrt{2x+5} - 3} = -\lim_{x \rightarrow 2} \frac{\sqrt{3x-2} - 1}{x - 2} \times \frac{x - 2}{\sqrt{2x+5} - 3}$$

$$= -\lim_{x \rightarrow 2} \frac{\sqrt{3x-2} - 1}{x - 2} \times \frac{1}{\frac{\sqrt{2x+5} - 3}{x - 2}} = -\lim_{x \rightarrow 2} \frac{u(x) - u(2)}{x - 2} \times \frac{1}{\frac{v(x) - v(2)}{x - 2}}$$

$$= -\lim_{x \rightarrow 2} \frac{u(x) - u(2)}{x - 2} \times \frac{1}{\frac{v(x) - v(2)}{x - 2}} = \dots$$

$$22. \lim_{x \rightarrow +\infty} \cos\left(\frac{x+1}{x+2}\right) = \cos 1$$

$$23. \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x}} = 0$$

$$24. \lim_{x \rightarrow 1} \cos \pi x + \frac{1}{x-1} = -1$$

$$25. \lim_{x \rightarrow \frac{\pi}{2}} \tan x - \frac{2}{x} = +\infty$$

$$26. \lim_{x \rightarrow 0} \frac{\sin x}{3x} = \frac{1}{3} \times \underbrace{\lim_{x \rightarrow 0} \frac{\sin x}{x}}_1 = \frac{1}{3}$$

$$27. \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x}} = \lim_{x \rightarrow 0} \sqrt{x} \times \frac{\sin x}{x} = 0$$

$$28. \lim_{x \rightarrow 0} \frac{x}{\tan 2x} = \frac{1}{2} \times \lim_{x \rightarrow 0} \frac{1}{\frac{\tan 2x}{2x}} = \frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$$

$$29. \lim_{x \rightarrow 0} \frac{\sin x}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{1}{x} \times \frac{\sin x}{x} \times \frac{x^2}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{1}{x} \times \frac{\sin x}{x} \times \frac{1}{\frac{1 - \cos x}{x^2}} = +\infty$$

$$30. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{\frac{\sin x - \cos x \sin x}{\cos x}}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{x^3 \cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \frac{1 - \cos x}{x^2} \times \frac{1}{\cos x} = 1 \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$$

$$31. \lim_{x \rightarrow 0} \frac{\sin 2x}{\sqrt{1 - \cos x}} = \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times \frac{2x}{\sqrt{2 \sin^2 \frac{x}{2}}} = \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times \frac{2x}{\sqrt{2} \sin \frac{x}{2}}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times \frac{4 \frac{x}{2}}{\sqrt{2} \sin \frac{x}{2}} = \frac{4}{\sqrt{2}} \times \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times \frac{\frac{x}{2}}{\sin \frac{x}{2}} = \frac{4}{\sqrt{2}} \times \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times \frac{1}{\frac{\sin \frac{x}{2}}{\frac{x}{2}}} = 2\sqrt{2}$$