EXERCICES CORRIGES : LIMITES ET CONTINUITE	
objectifs	Limites trigonométriques
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Exercice

Calculer les limites suivantes

a) $\lim\limits_{x\to 0}\dfrac{1+\sin x-\cos x}{1-\sin x-\cos x}$

b) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1+\sin x-\cos x}{1-\sin x-\cos x}$

c) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1+\tan x}{1-\tan x}$

d) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\cos x}{1-\sin x}$

Corrigé

a) $\lim\limits_{x\to 0}\dfrac{1+\sin x-\cos x}{1-\sin x-\cos x}$

Transformons les expressions

$$1+\sin x-\cos x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}$$

$$=2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}=2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)$$

$$1-\sin x-\cos x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}$$

$$=2\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}=2\sin\frac{x}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)$$

$$\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=\lim_{x\to 0}\frac{2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{2\sin\frac{x}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}=\lim_{x\to 0}\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}-\cos\frac{x}{2}}=-1$$

b) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1+\sin x-\cos x}{1-\sin x-\cos x}=\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}-\cos\frac{x}{2}}=1$

c) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1+\tan x}{1-\tan x}$

Posons $X=\tan x$, quand $x\to\dfrac{\pi}{2}$, $X\to+\infty$

Ainsi, $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1+\tan x}{1-\tan x}=\lim\limits_{X\to+\infty}\dfrac{1+X}{1-X}=-1$

d) $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos x}{1-\sin x}$

Posons $t=\tan\dfrac{x}{2}$, ainsi quand $x\to\dfrac{\pi}{2}$, $t\to1$

$$\cos x=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}=\frac{\dfrac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}}-\dfrac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}{\dfrac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}}+\dfrac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}=\frac{1-t^2}{1+t^2}$$

$$\cos x=\frac{1-t^2}{1+t^2}$$

$$\sin x=\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}=\frac{\dfrac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}}}{\dfrac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}}+\dfrac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}=\frac{2t}{1+t^2}$$

$$\sin x=\frac{2t}{1+t^2}$$

$$1-\sin x=1-\frac{2t}{1+t^2}=\frac{1+t^2-2t}{1+t^2}=\frac{(t-1)^2}{1+t^2}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{1-\sin x}=\lim_{t\to1}\frac{1-t^2}{1+t^2}\times\frac{1+t^2}{(1-t)^2}=\lim_{t\to1}\frac{1-t^2}{(1-t)^2}=\lim_{t\to1}\frac{1+t}{1-t}=+\infty$$

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